ISING MACHINES

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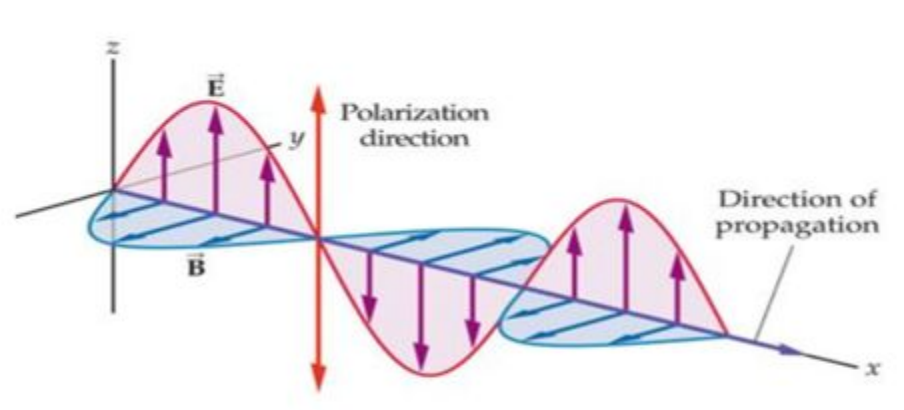
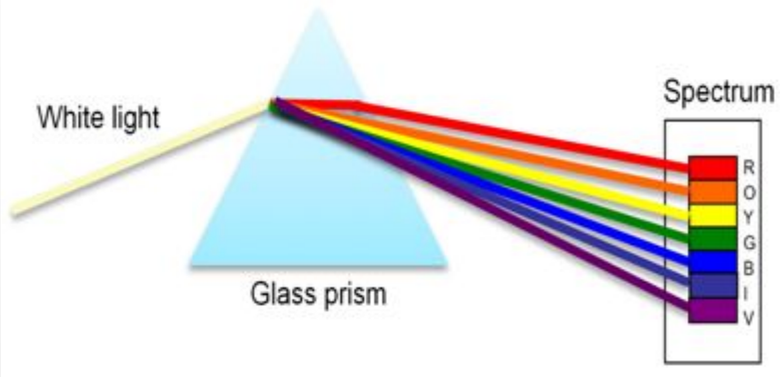
Outline

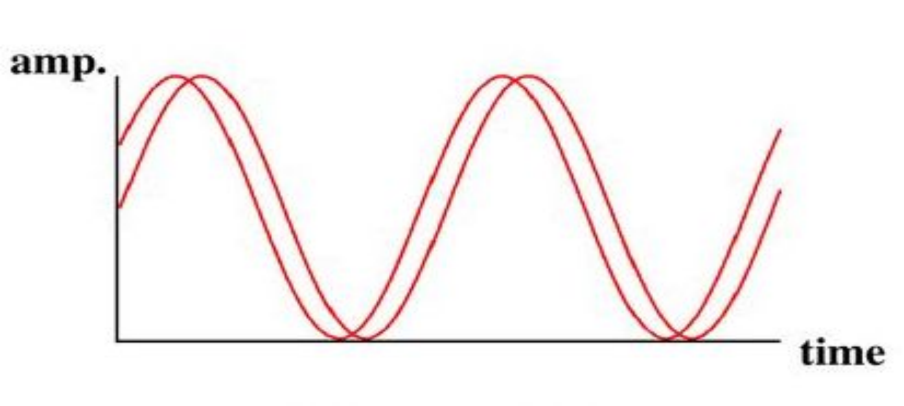
- Some properties of light
  - Superposition, Entanglement
  - Wave – Particle Duality
  - Classical versus Quantum Experiments
- Photonic Ising Machines
  - Multiplexed in Time – Poor Man's Ising Machine
  - Multiplexed in Space – Spatial Light Modulation
- Chip-based Ising Computing Machines
  - Electronic oscillators
  - Magnetic oscillators

Properties of Light



This wave **is polarized** in z -direction





- Polarization
- Color
  - Wavelength, Frequency
- Phase
- Spatial Modes

How does Quantum Computing work?

- Classical bits versus quantum qubits

A	B
0	0
0	1
1	0
1	1

2 bits of information
State of A and of B

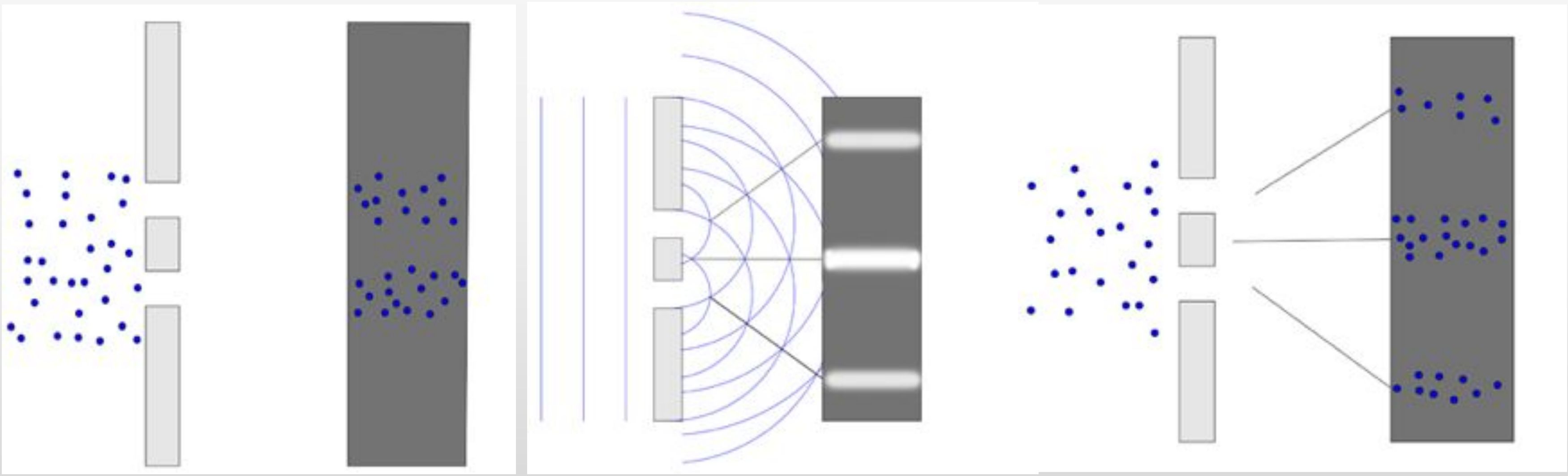
A	B	probability
0	0	α
0	1	β
1	0	γ
1	1	δ

4 bits of information
Superposition state

Waves interfere, we use them for computation

- Extend this to 3 qubits – 8 bits of information
- n-qubits will have 2^n bits of information

How does Photonic Computing work?

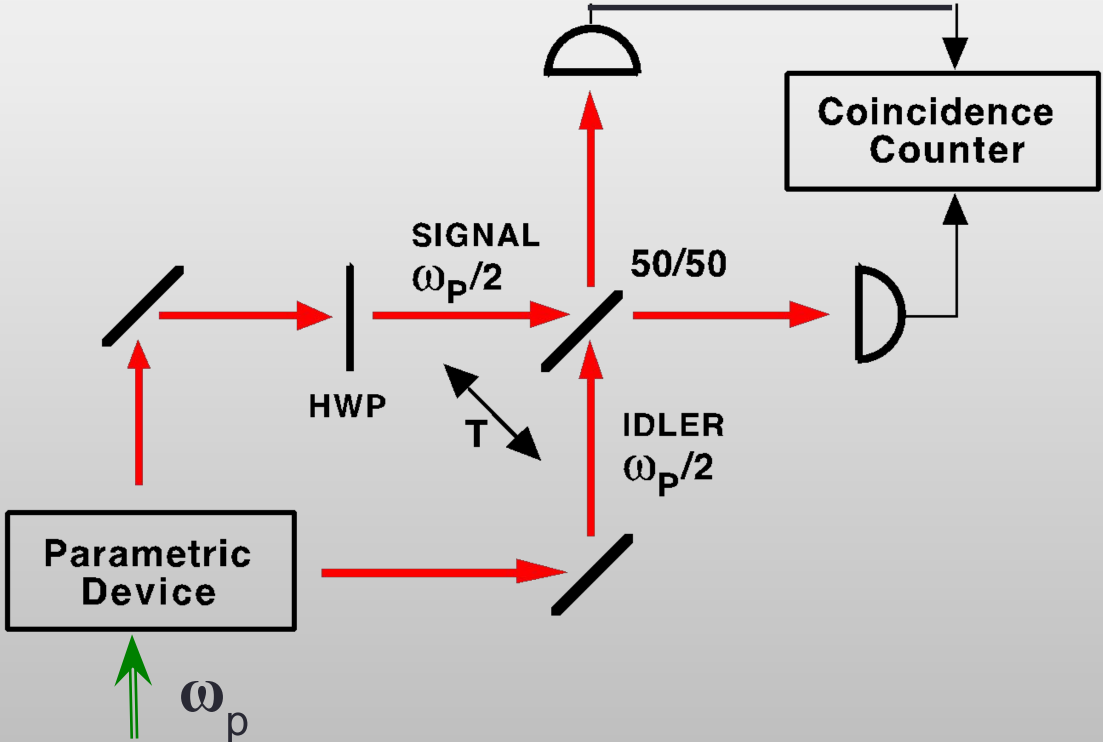


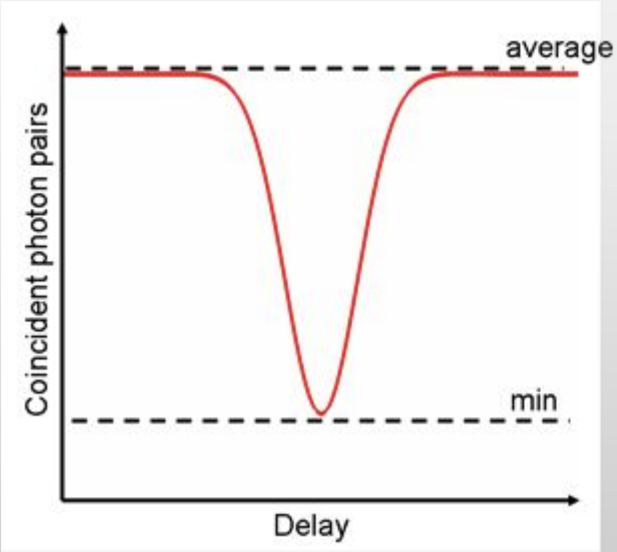
(a) Particles (b) Waves (c) Particles as Waves

- Double slit experiment – wave particle duality
- Analogy – massively parallel computation

Testing a Single Photon Source

- Hong-Ou-Mandel Interferometer

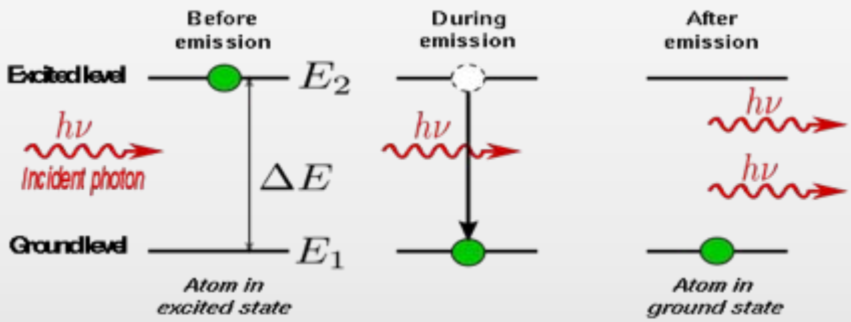




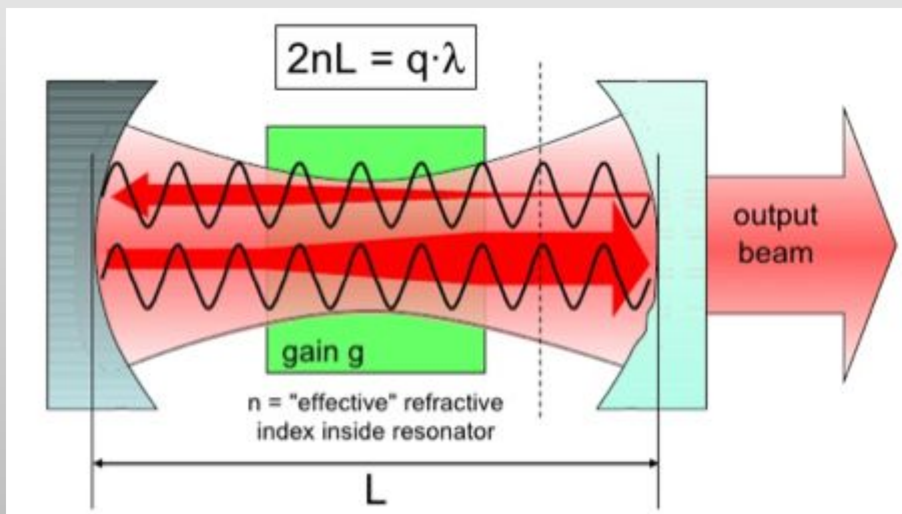
HOM dip as a function of T

Coincidence of counts shows a non-classical dip

How does a laser work?



$$E_2 - E_1 = \Delta E = h\nu$$



Stimulated Emission is in phase with the incoming photon

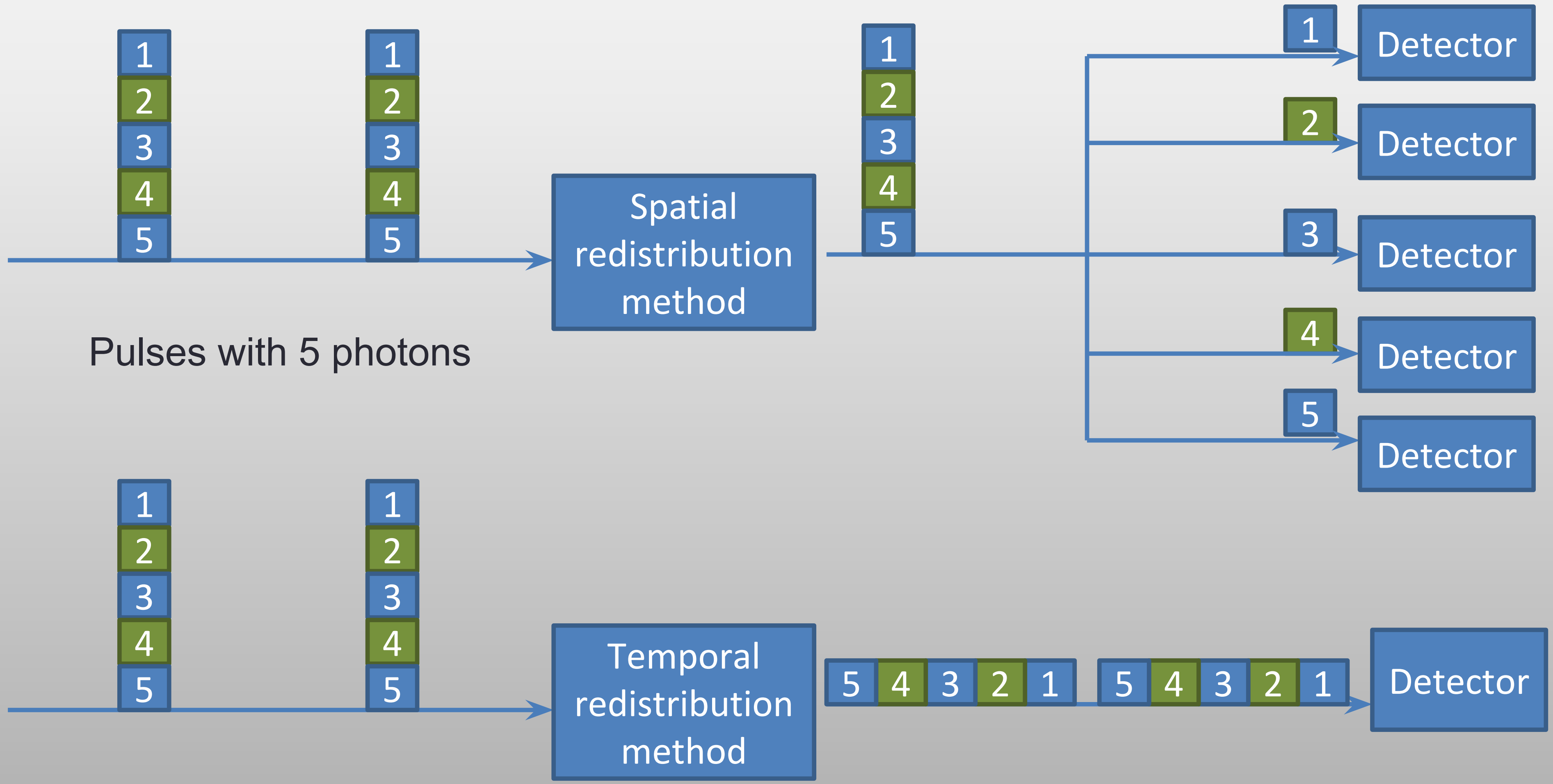
A laser is Coherent. All the photons are locked in phase.

Can measure a coherence time.

- Optical wave (photons) oscillating inside a resonant cavity

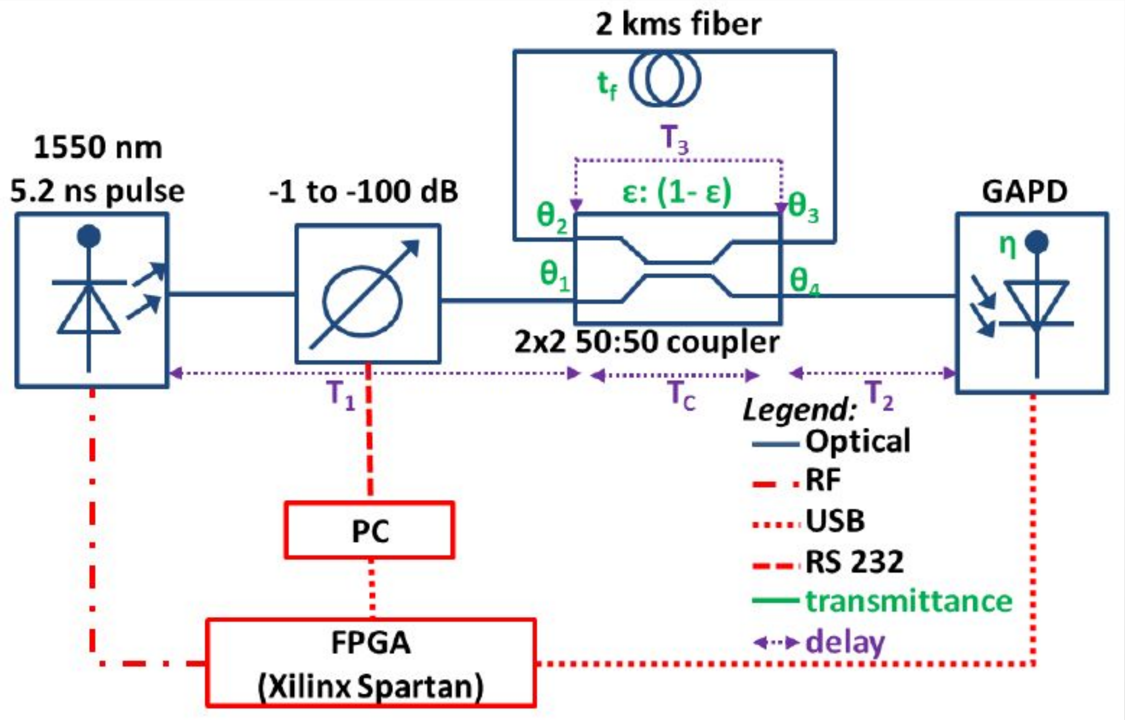
How do we resolve photon numbers?

- Redistribution into different spatial or temporal bins

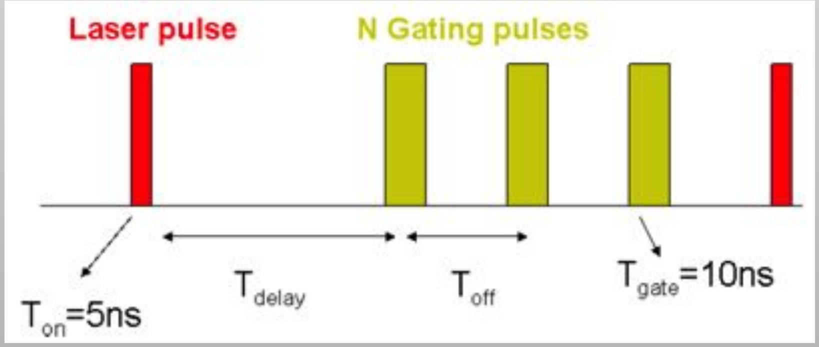


Temporal redistribution

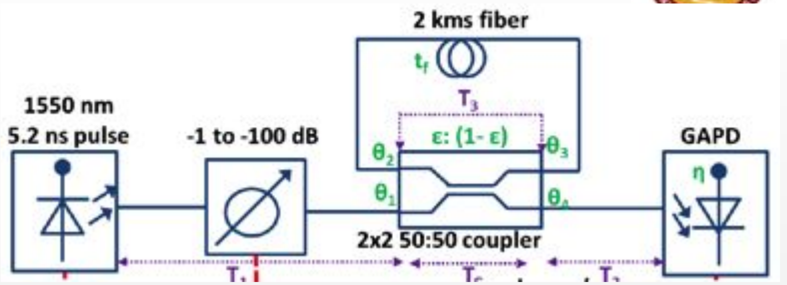
- Average power reduced by half for each circulation

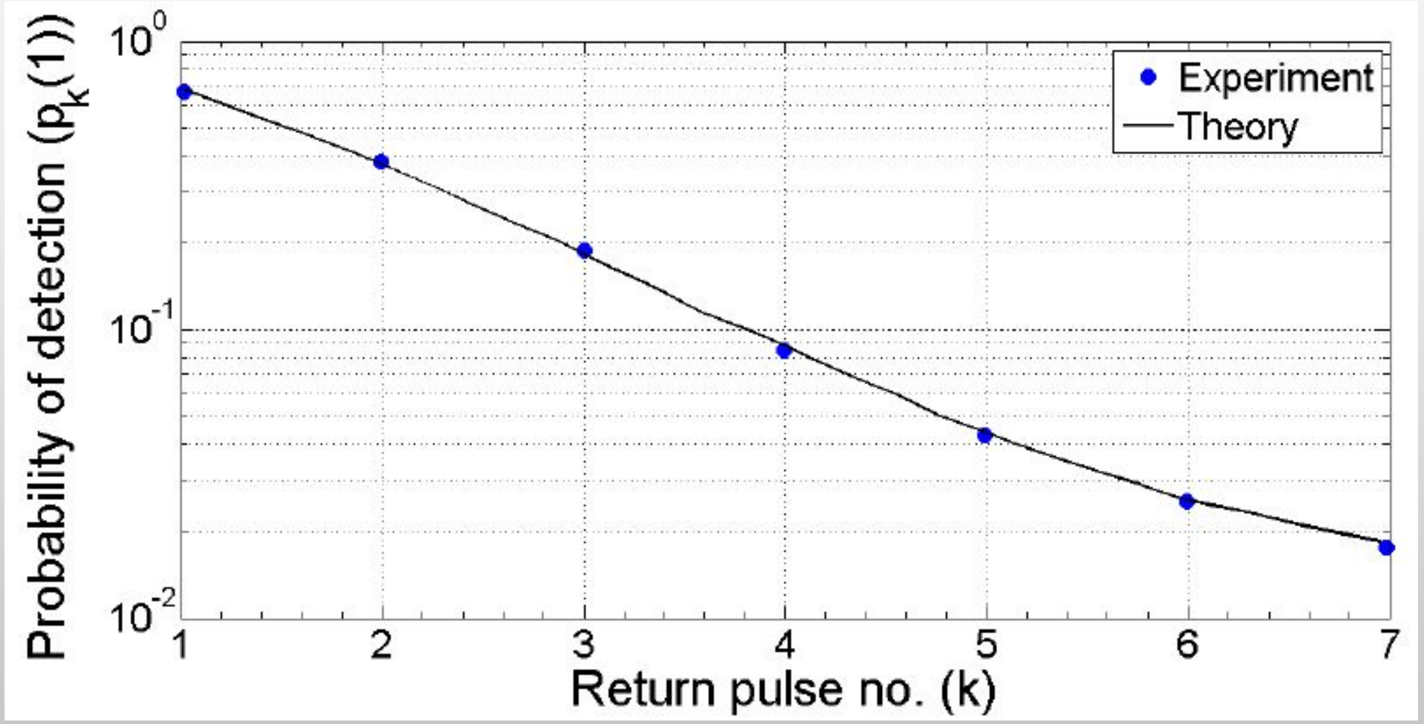


- Synchronized detection using gating at GAPD



Detection probabilities

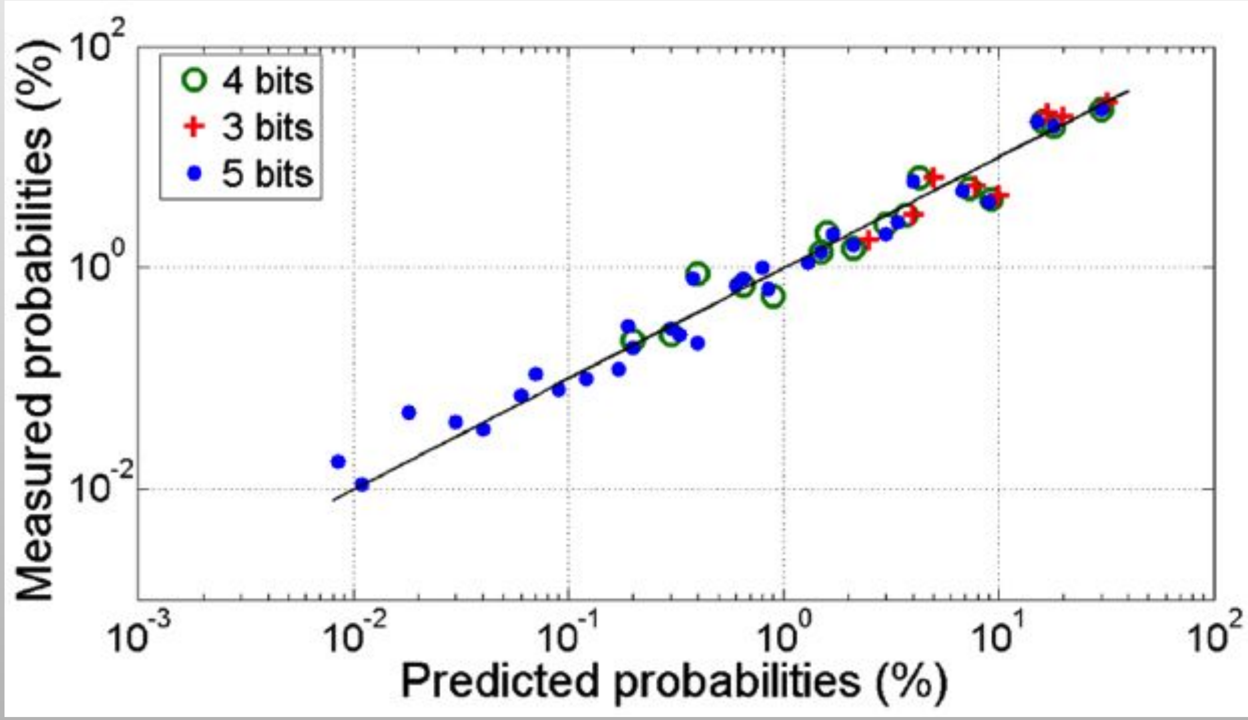




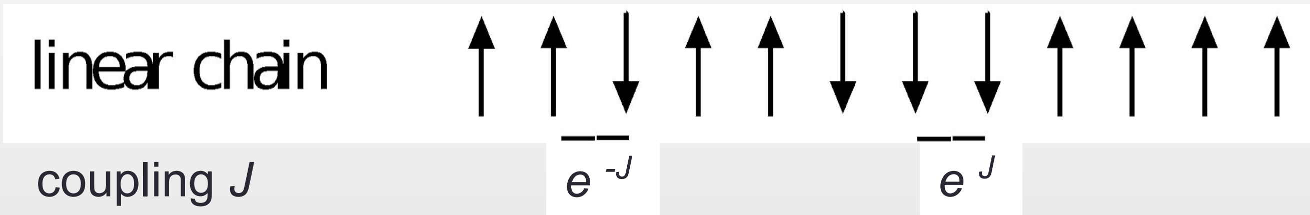
- Decreasing detection probabilities with subsequent redistribution

Predicting detections

- If it is classical, we can predict patterns
  - 10…, 11
  - 100…, 101, 110, 111…
  - 1000, 1001, 1010, 1011 … etc



The Ising Model

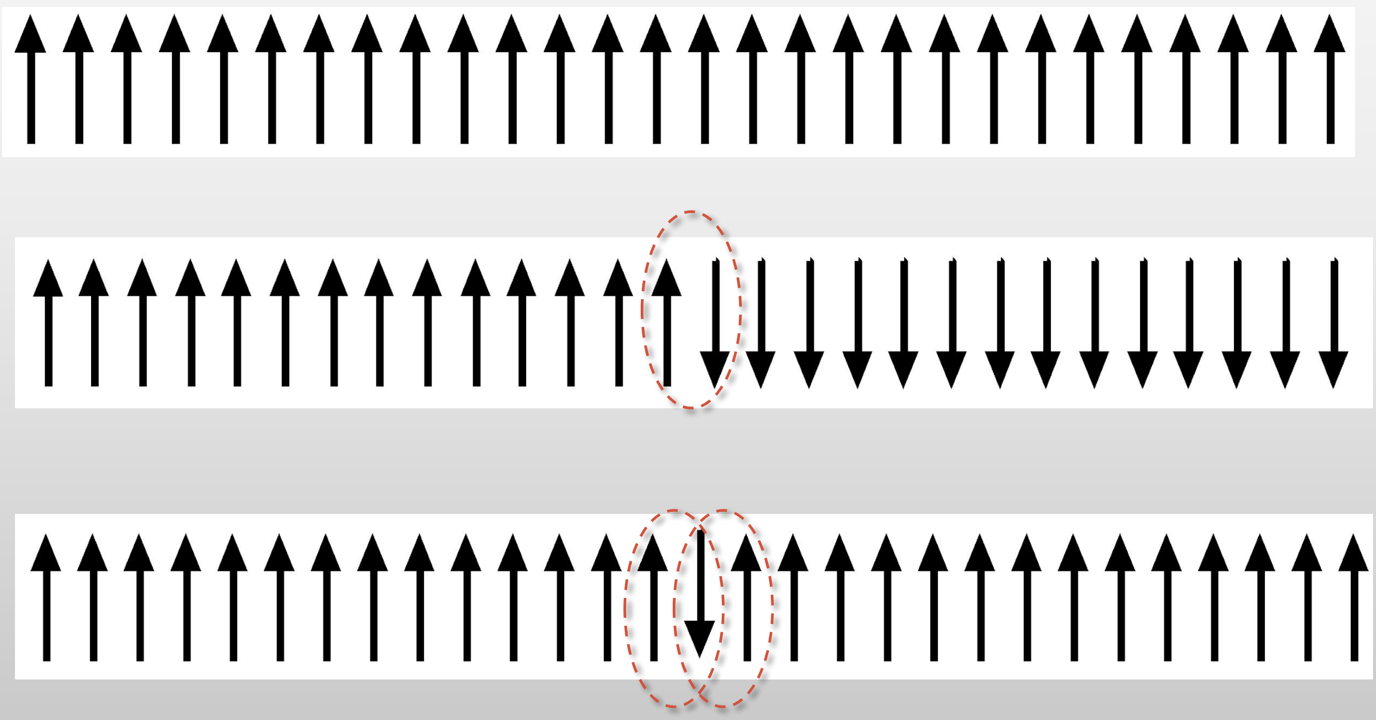


- Popular model in Statistical Physics
- Phase Transition
- Percolation Theory
- Ferromagnetic vs Antiferromagnetic

$$H_{\text{Ising}} = -\frac{1}{2}\sum_{mn}^{N} J_{mn}\sigma_m\sigma_n.$$

$$J = \begin{cases} > 0 & \text{ferromagnetic} \quad \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \\ < 0 & \text{antiferromagnetic} \quad \downarrow\uparrow\downarrow\uparrow\downarrow\uparrow \end{cases}$$

Bloch Wall



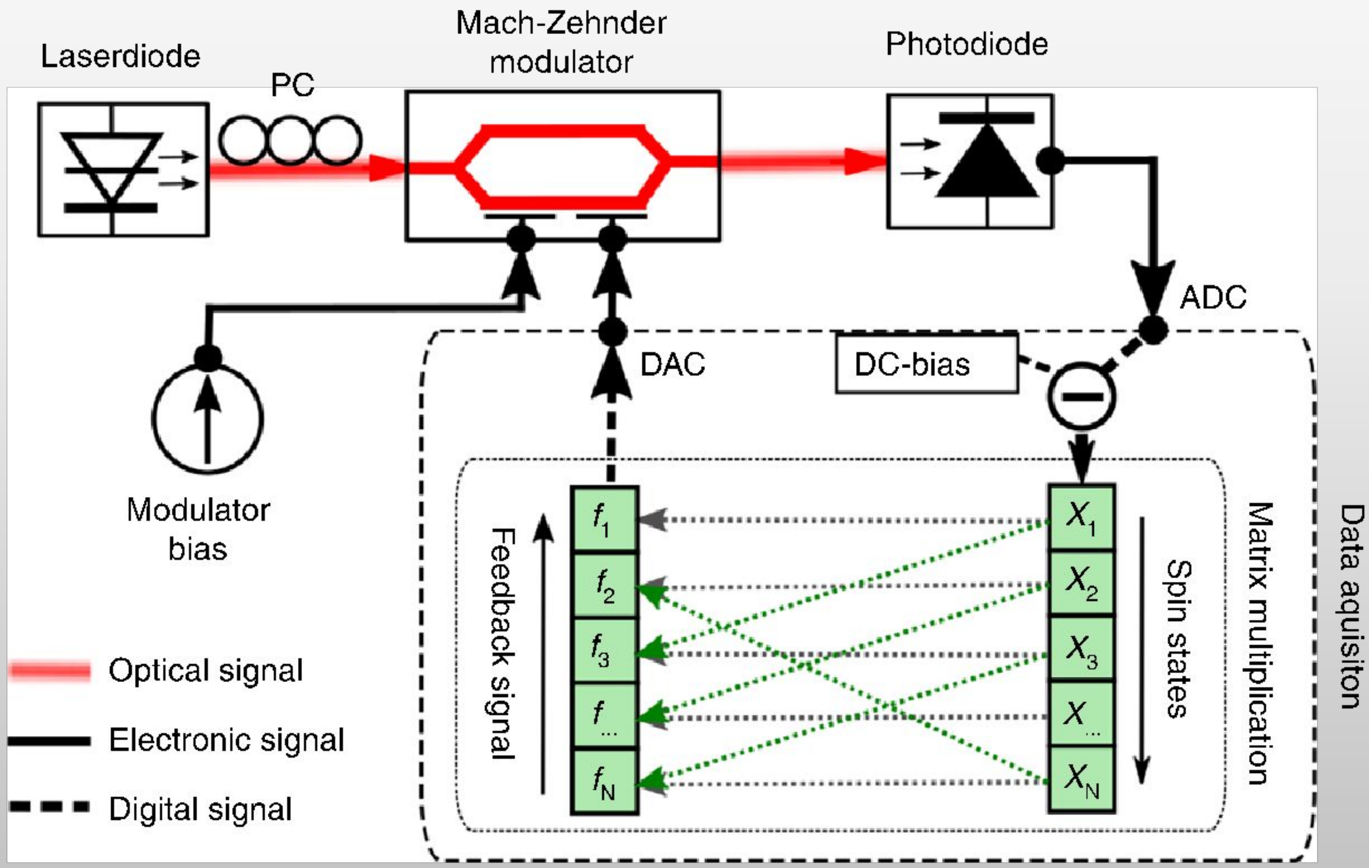
$E = 0$

$E = e^{-J}$

$E = e^{-2J}$

- Spins will orient themselves based on the minimum energy configuration
- Costs less energy to create a Wall than to flip one spin

The Poor Man’s Ising Machine



- Mach Zender Modulator with optoelectronic feedback
- Bohm et al, Nature Communications, 10:3538,2019.

The Optical-Electronic-Optical Model

$$x_n[k+1] = \cos^2(f_n[k] - \pi/4 + \zeta_n[k]) - \frac{1}{2}.$$

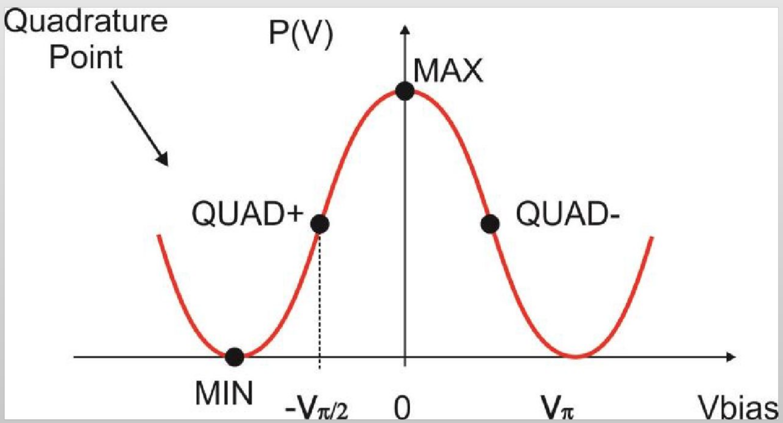
$$f_n[k] = \alpha x_n[k] + \beta \sum_m J_{mn} x_m[k].$$

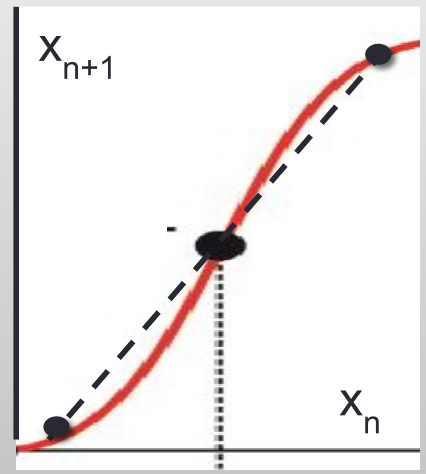
$$\sigma_n = sig_n(x_n[k])$$

- Self bias term α
- Coupling coefficient β
- Weights between spins J
- What does the optics do?
  - Nonlinear function – $\cos^2$

MZM Transfer Function

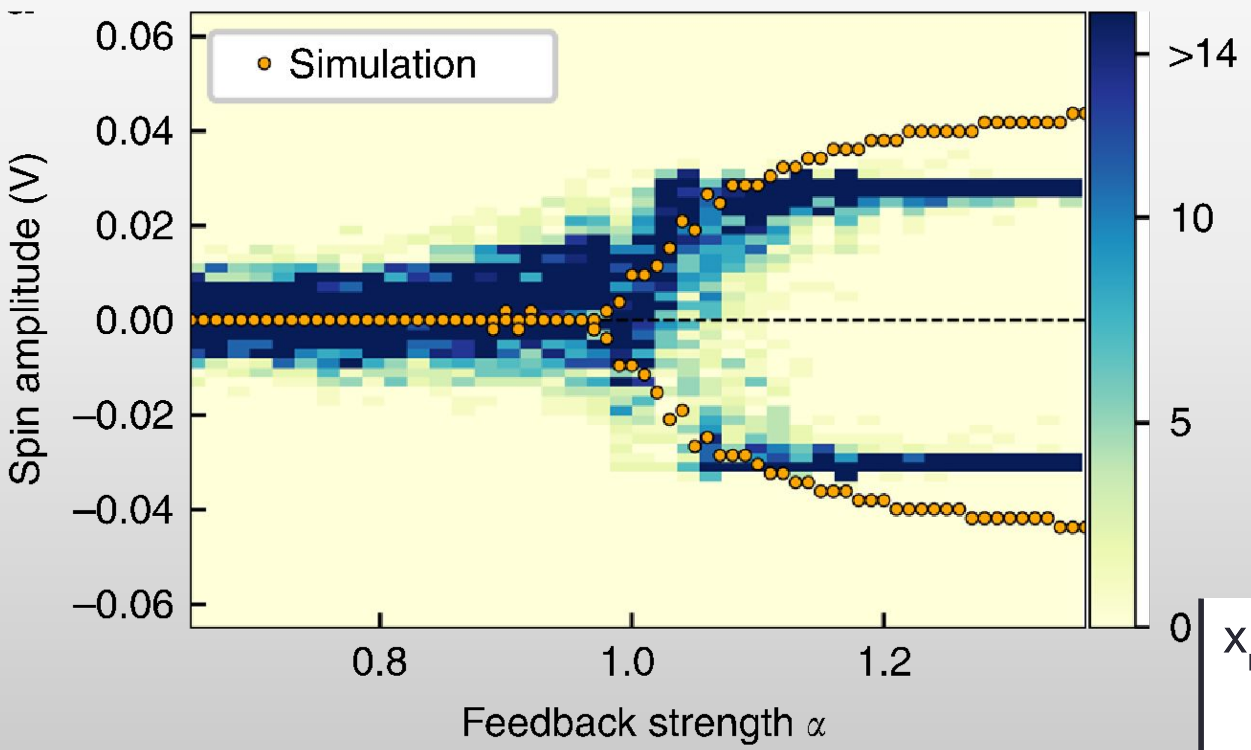
$$I_{out}(t) = T_{mod}\frac{I_{in}}{2}\left[1 + cos\left(\frac{\pi}{V_{\pi}}V(t) - \phi\right)\right]$$

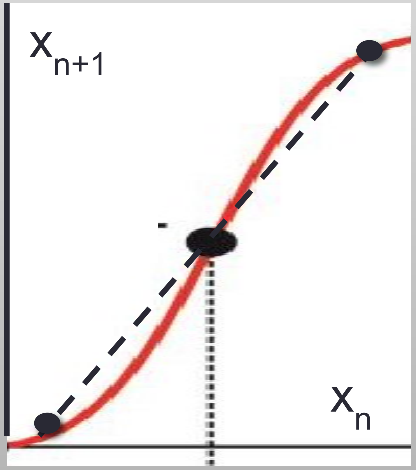




- Find the correct bias point, so we have two solutions - bifurcation

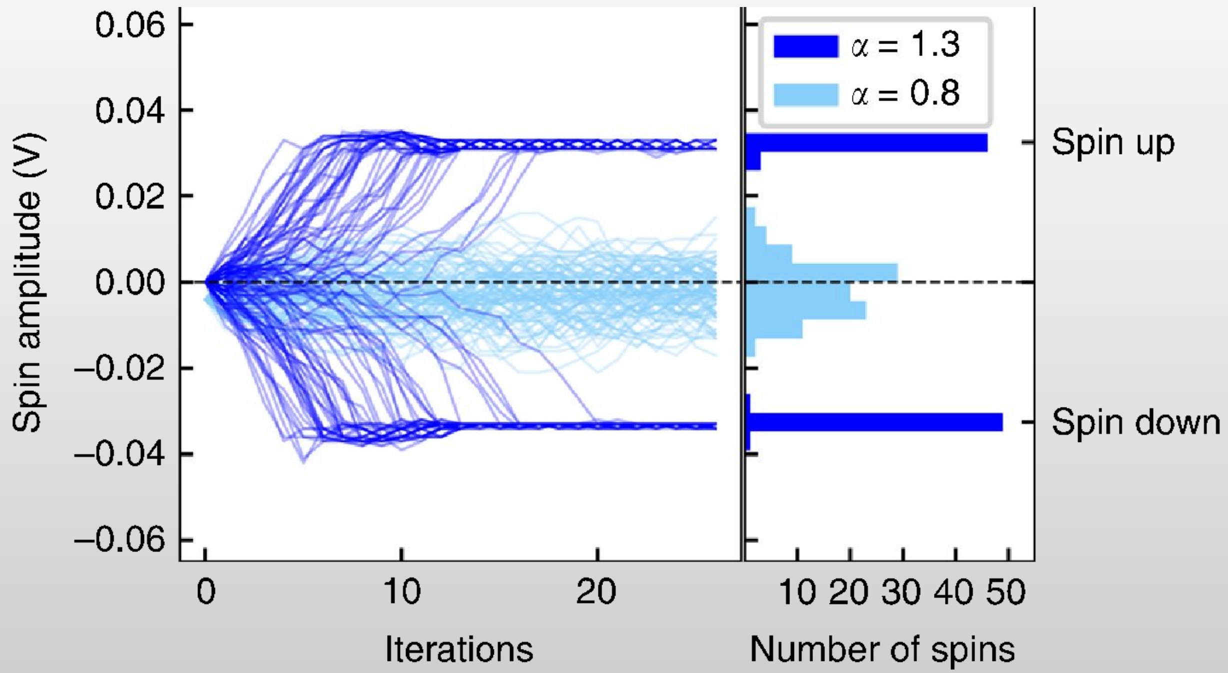
Pitchfork Bifurcation





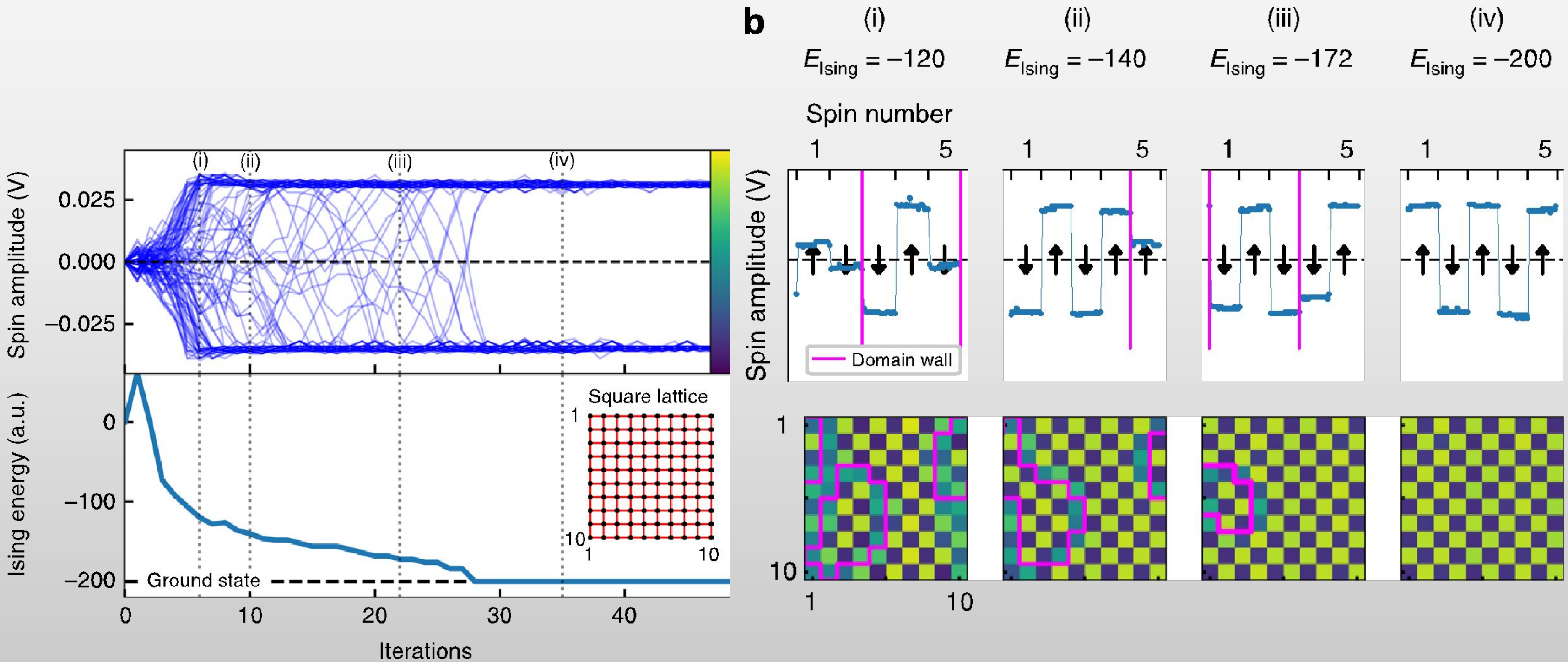
- Nonlinear function has two solutions
- Uncoupled spins

Pitchfork Bifurcation



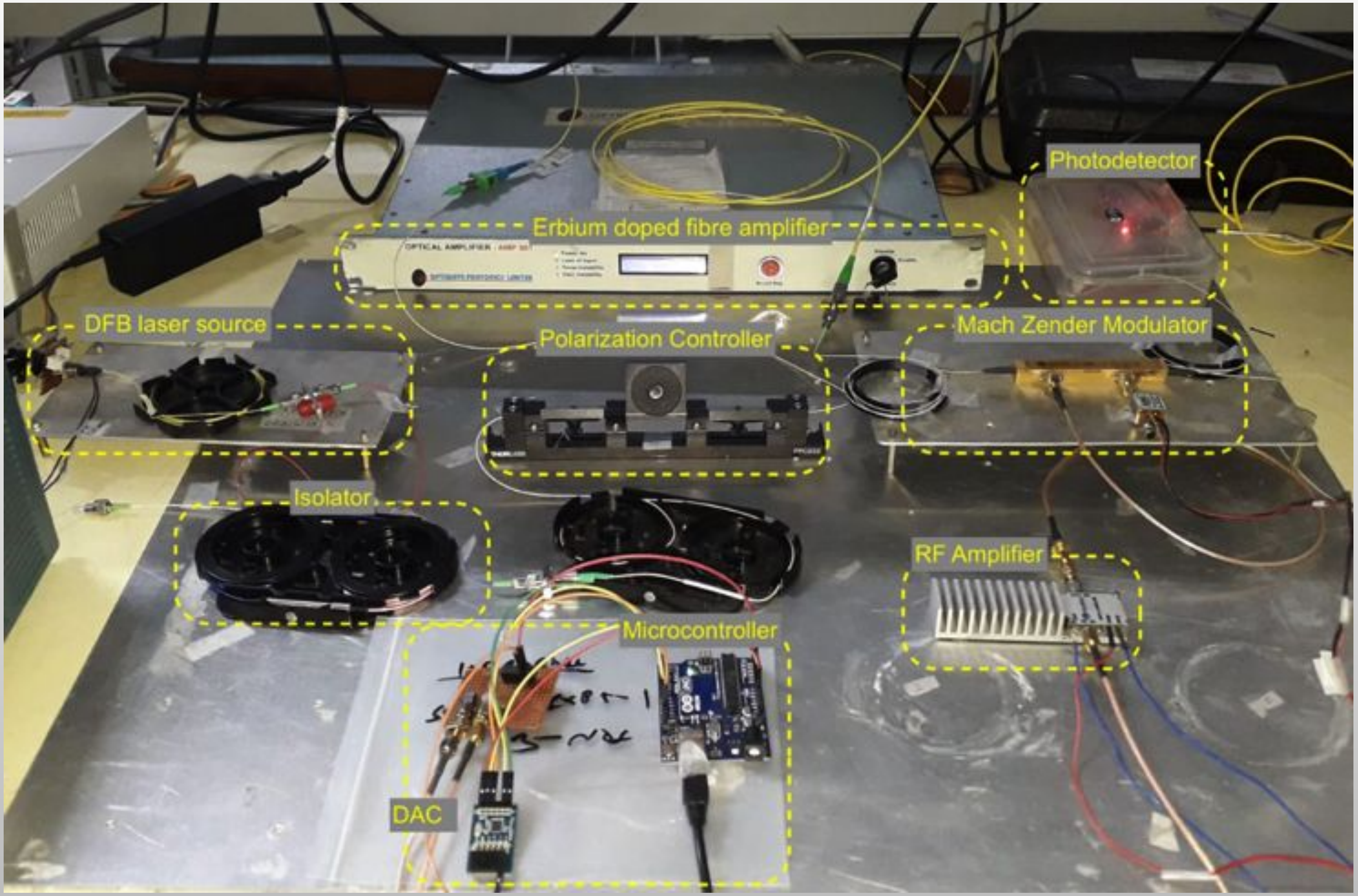
- With 100 spins
- Tune α to observe the bifurcation

Square Lattice



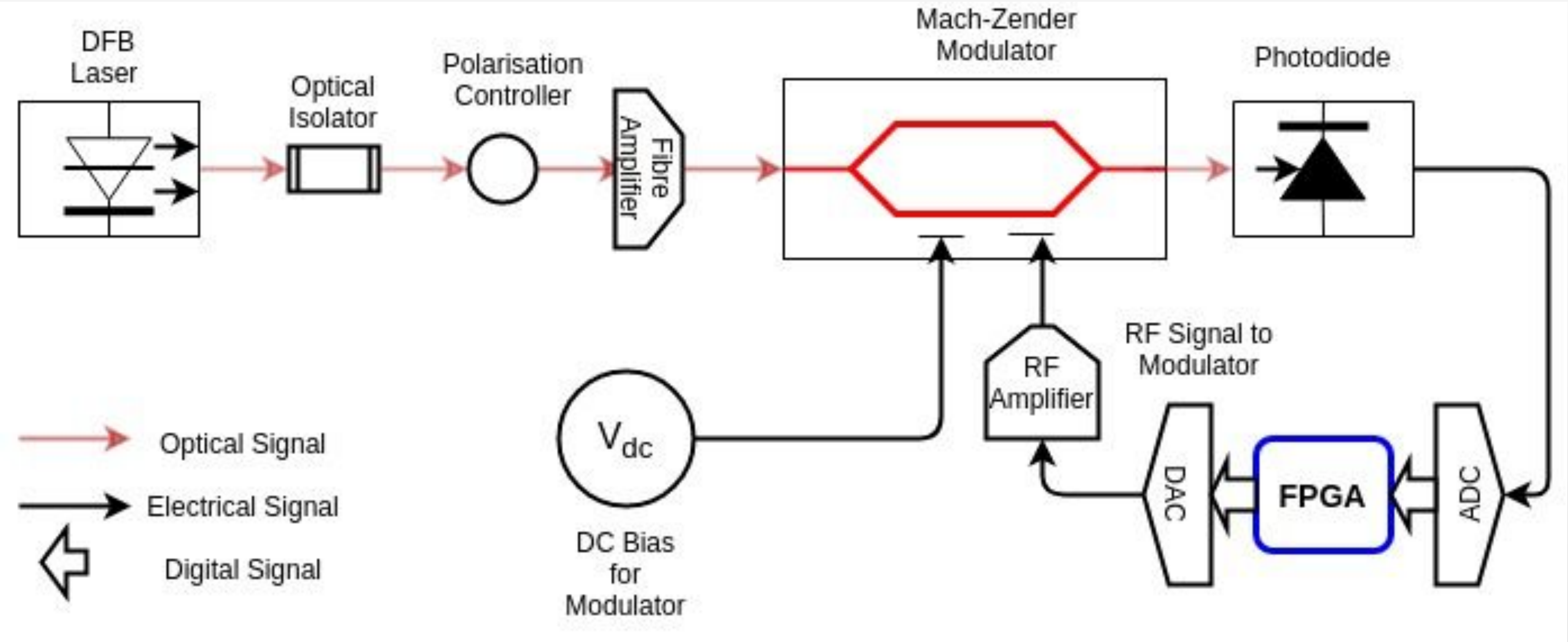
- 10 x 10 lattice, α = 0.25, β = 0.29
- Can observe domain walls where spins are aligned up
- Lowest energy for the checkerboard pattern

The Poor Man's Ising Machine



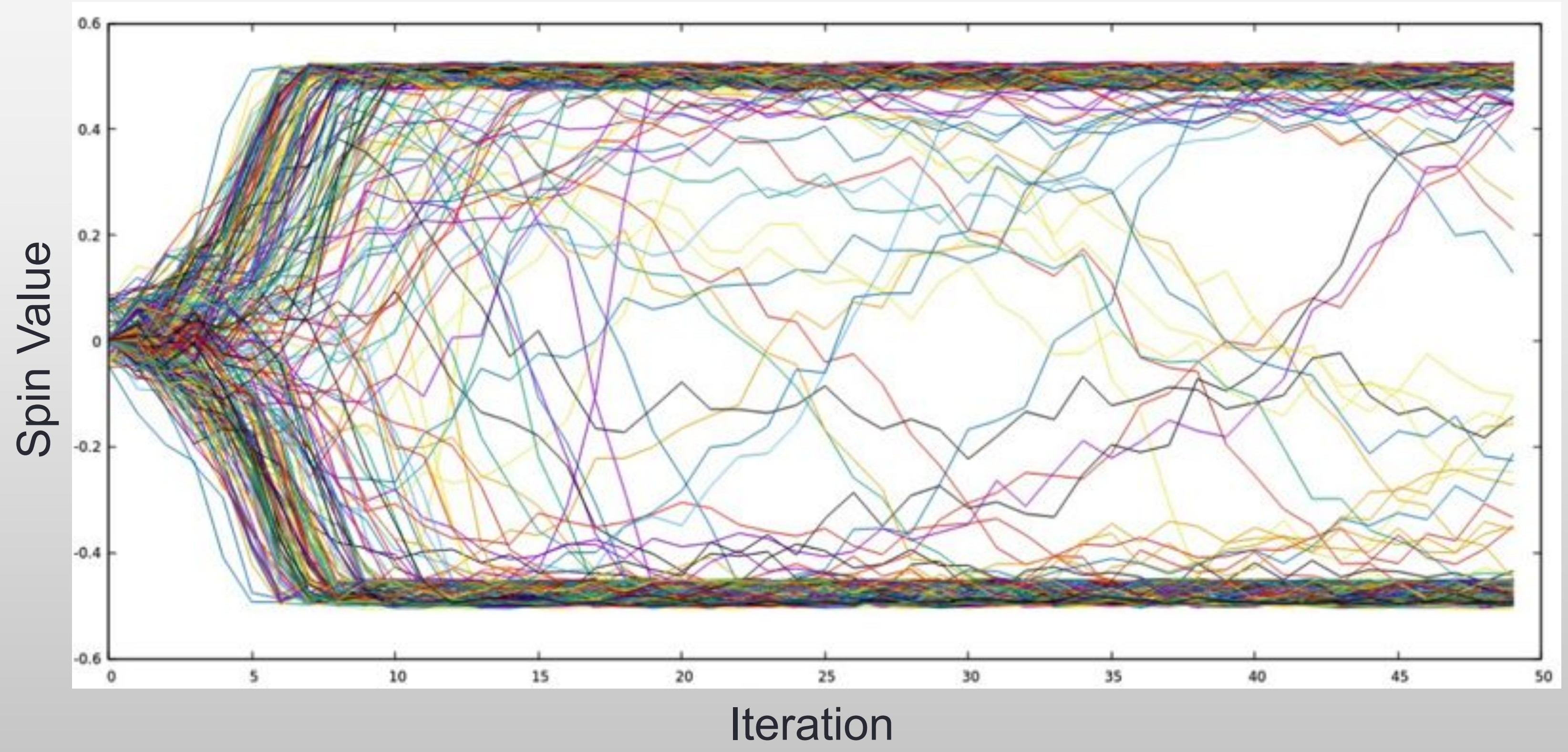
- Not quite Poor….about $10,000

The Poor Man’s Ising Machine



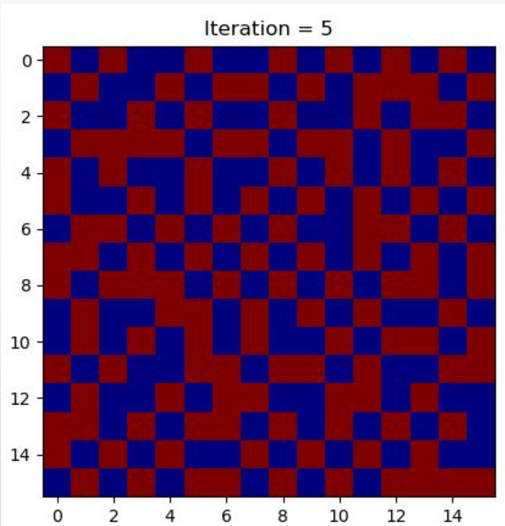
- It is actually a little more complicated
- Needed an optical isolator, a polarization controller, and an optical amplifier

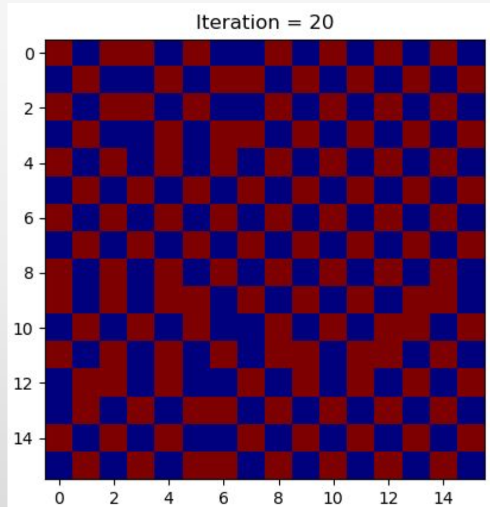
16 x 16 spin lattice

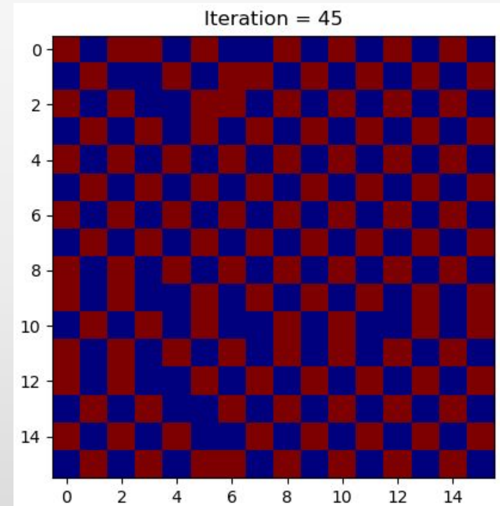


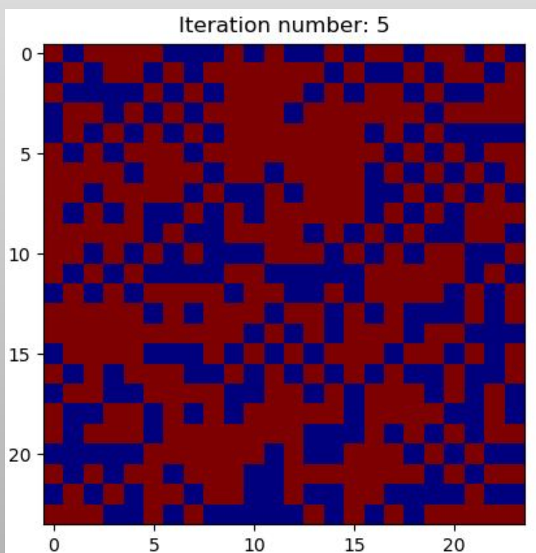
- Nearest Neighbours on a square lattice
- Gautham and Parth, IIT Madras

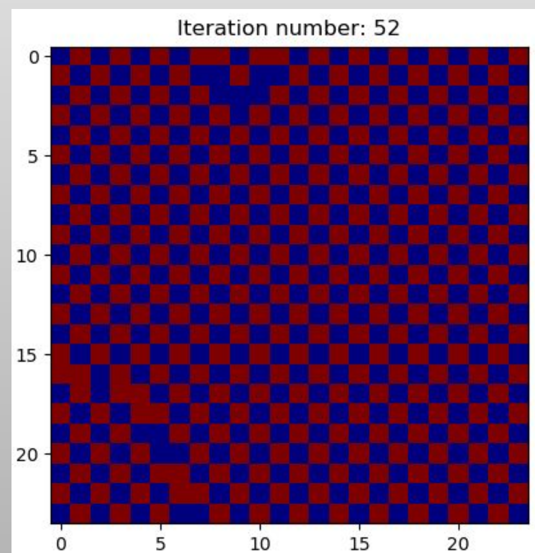
Spin lattice (16x16, 24x24)

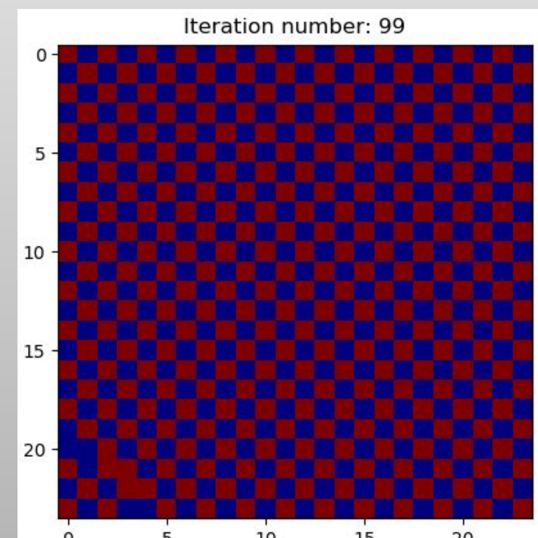




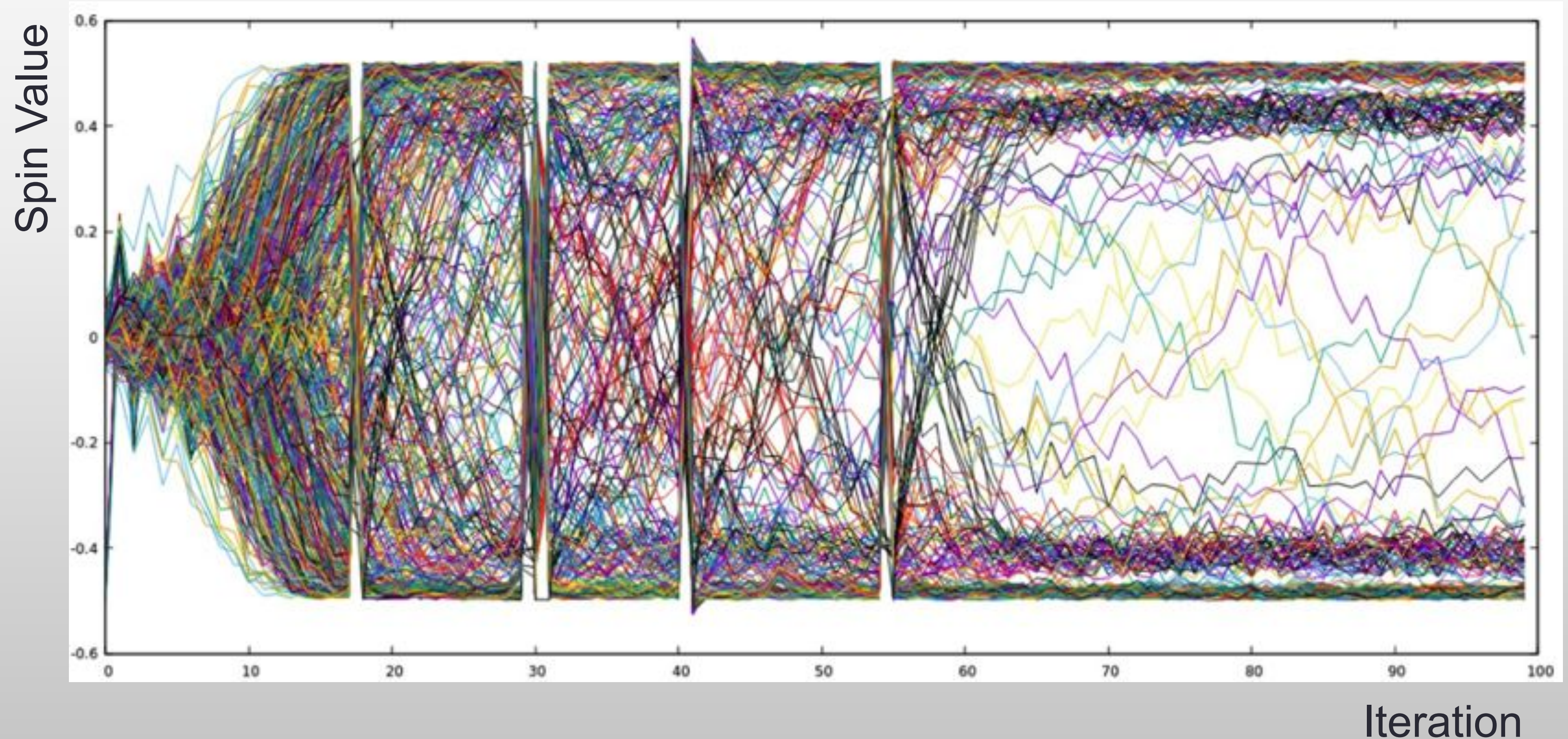








24 x 24 spin lattice



- Discontinuities are under investigation
- Gautham and Parth, IIT Madras

How well does it scale?

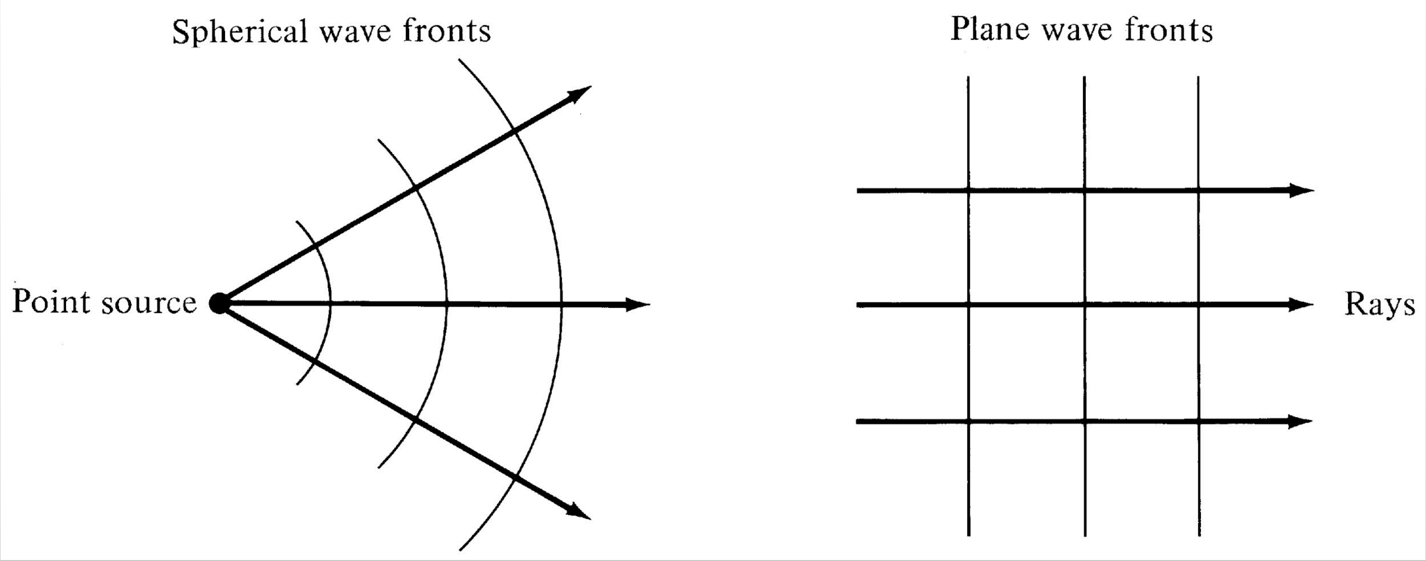
- Photonic Chips, NUS,Singapore

Spatial Light Ising Machine (SLIM)

Large-Scale Photonic Ising Machine by Spatial Light Modulation

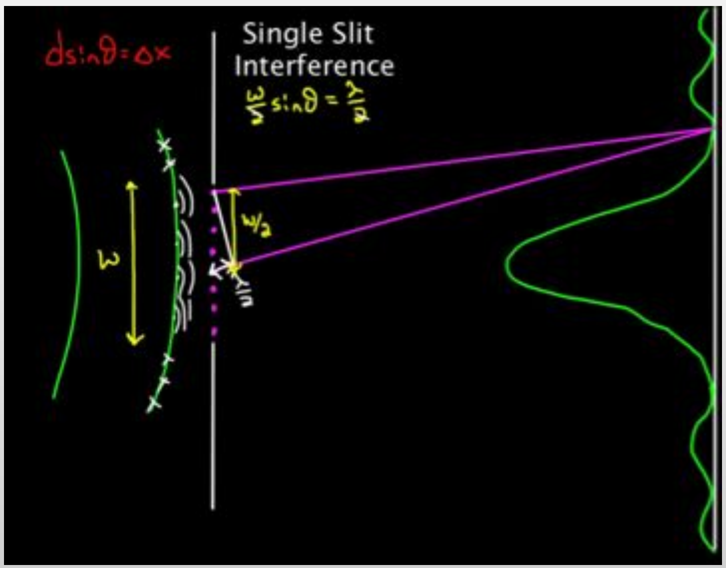
PHYSICAL REVIEW LETTERS **122,** 213902 (2019)

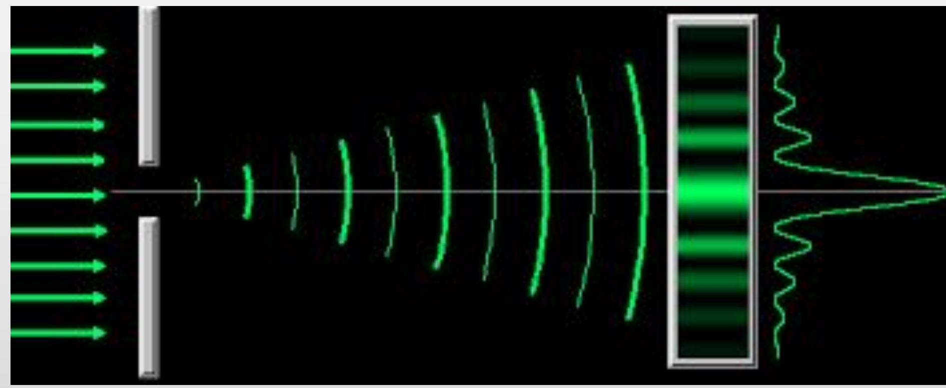
D. Pierangeli,[1,2,*] G. Marcucci,[1,2] and C. Conti[1,2]



- Optical beams have spatial divergence
- Interactions are between different rays - phase

Understanding Phase

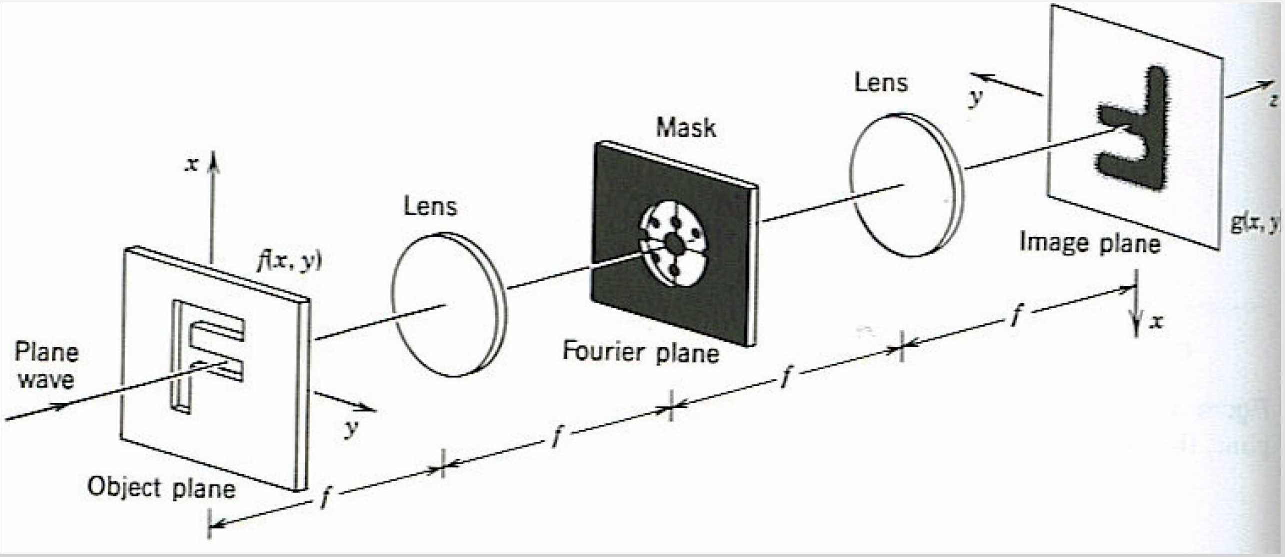




Diffraction from a single slit

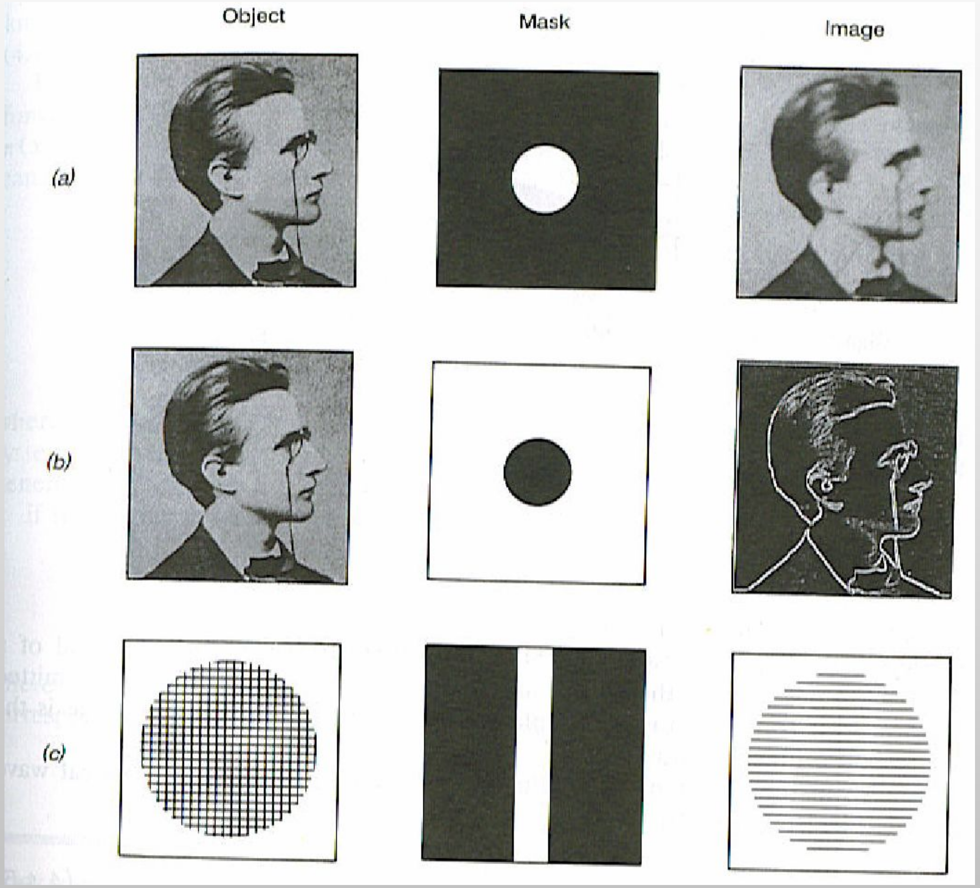
- Different rays of light accumulate a different phase based on their propagation distance from the source
- Think of diffraction in 2-D, you will see rings

Fourier Optics



- The 4-f system
- Every optical ray in 2-D interacts with every other ray
- A lens in 2-D acts as a Fourier Transformer, gives spatial frequency content

Spatial Filters



Blurred

Edge detection

- Easy to design high pass, low pass and vertical pass filters

Large-Scale Photonic Ising Machine by Spatial Light Modulation

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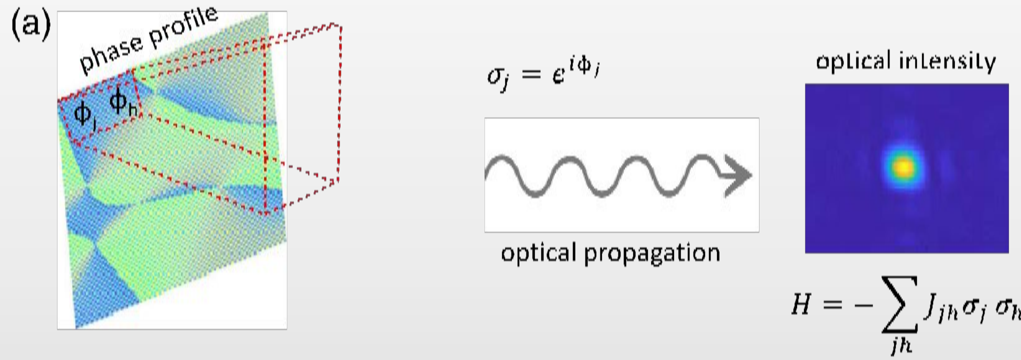
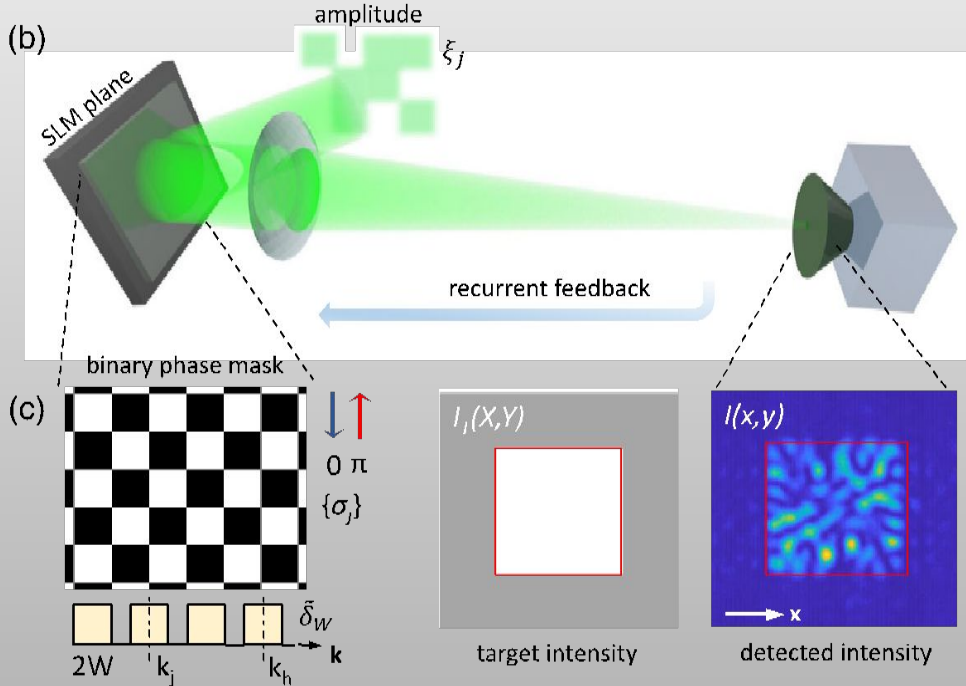
D. Pierangeli,[1,2,*] G. Marcucci,[1,2] and C. Conti[1,2]

- Amplitude mask
- Phase plane (SLM)
- Camera
- Feedback





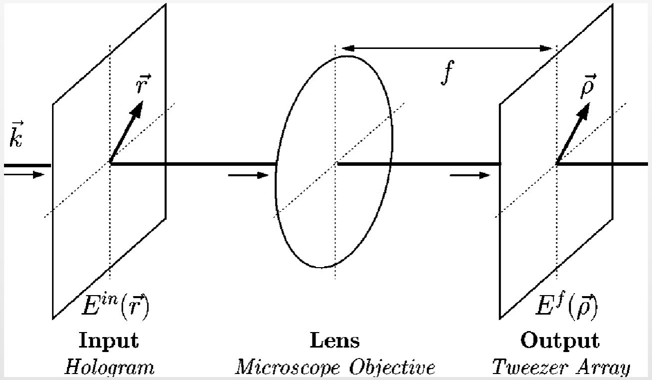
Far field intensity

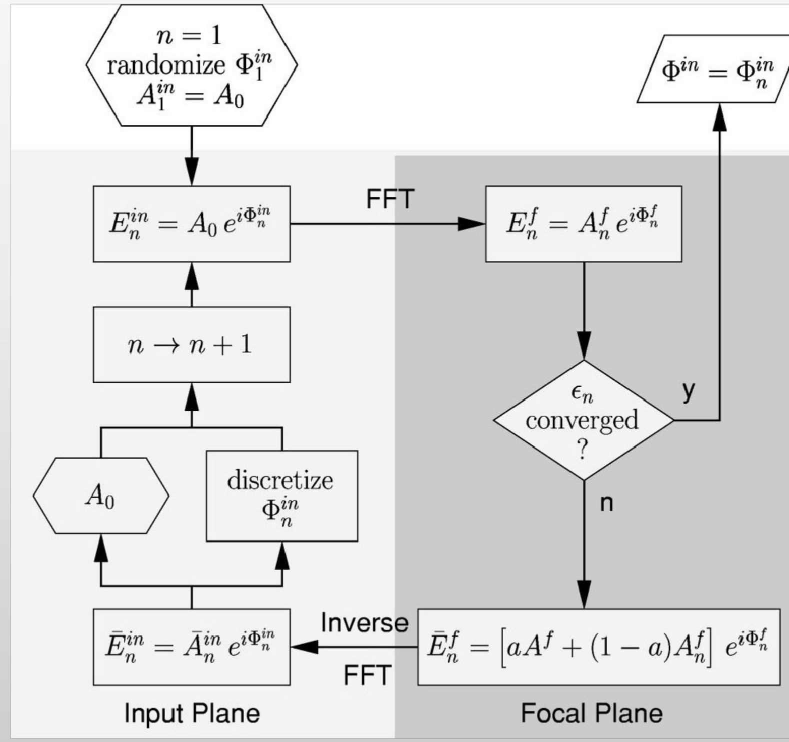
$$I(x) = |E(x)|^2 = \sum_{jh} \xi_j \xi_h \sigma_j \sigma_h \delta_W^2(x) e^{2\imath W(h-j)x}$$

Fourier transform of a rectangle

$$\delta_W(x) = \sin(Wx)/(Wx)$$

SLIM Algorithm





What are its limitations?

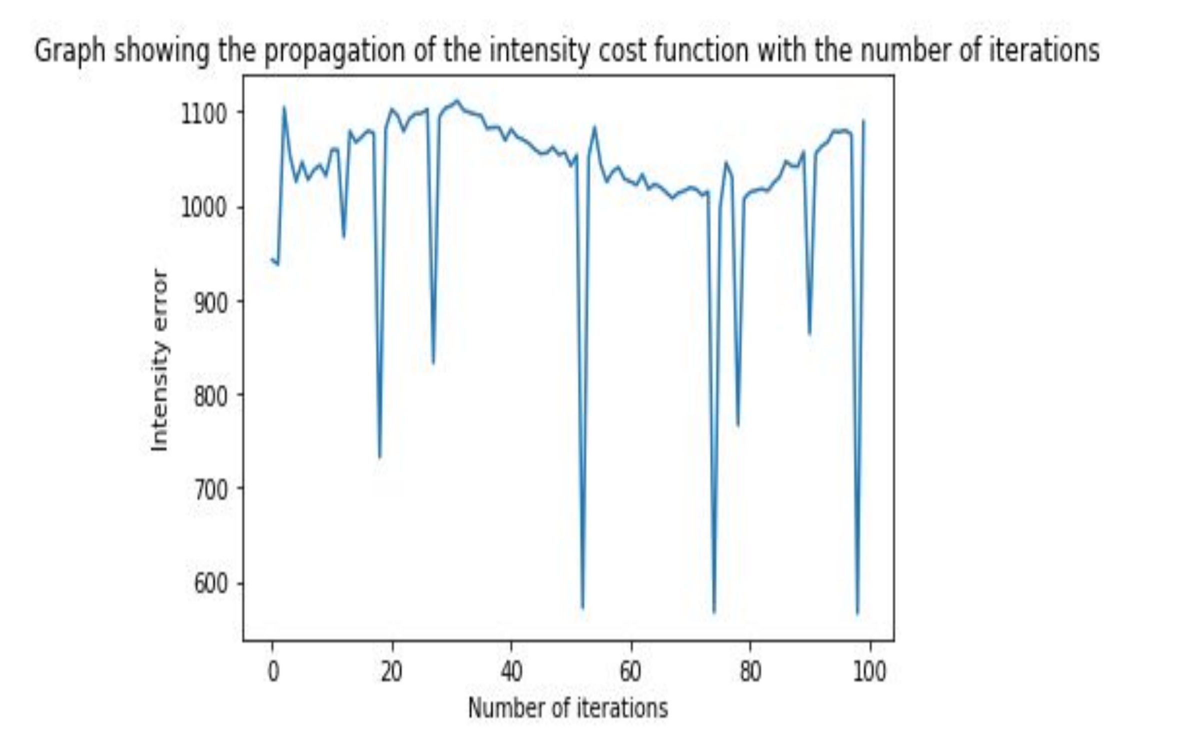
- We need *Φ(x,y)*
- Write an image to the SLM
- Read 1000 x 1000 pixels off a camera

$$Error = \sum_{x,y} \frac{[I_{target}(x,y) - I_{measured}(x,y)]^2}{No.\ of\ pixels}$$

- Computer-generated holographic optical tweezer arrays
- Dufresne et al, Rev. Sci. Instr., **72**, 2001

Convergence Issues

- We lose phase information with a camera
- SLM is slow and resets every so often
- 500 iterations takes 1 hour

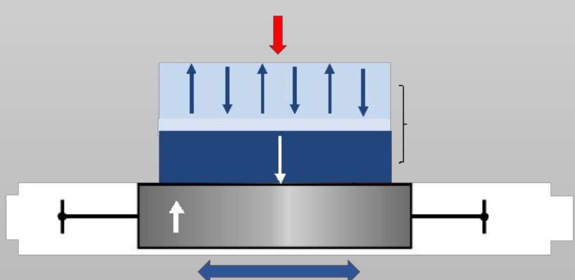


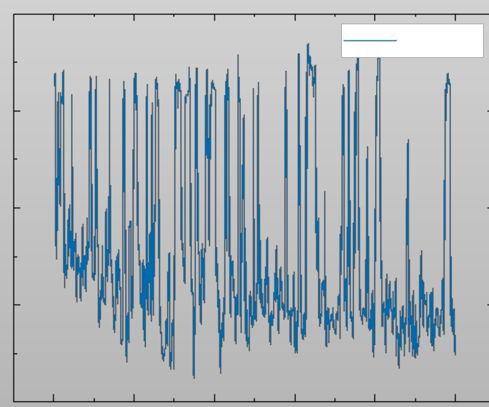
$$Error = \sum_{x,\,y} \frac{[I_{target}(x,y) - I_{measured}(x,y)]^2}{No.\ of\ pixels}$$

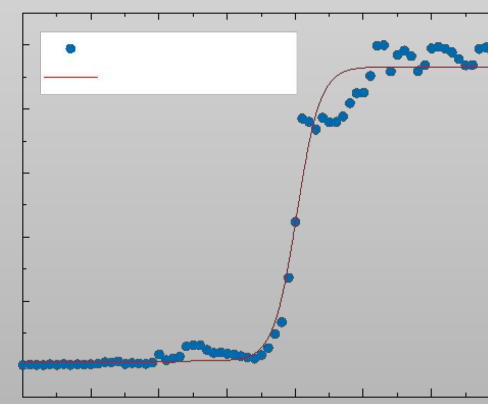
Chip based Ising Computing Machine

- CICM oscillators can be optical, magnetic or electronic

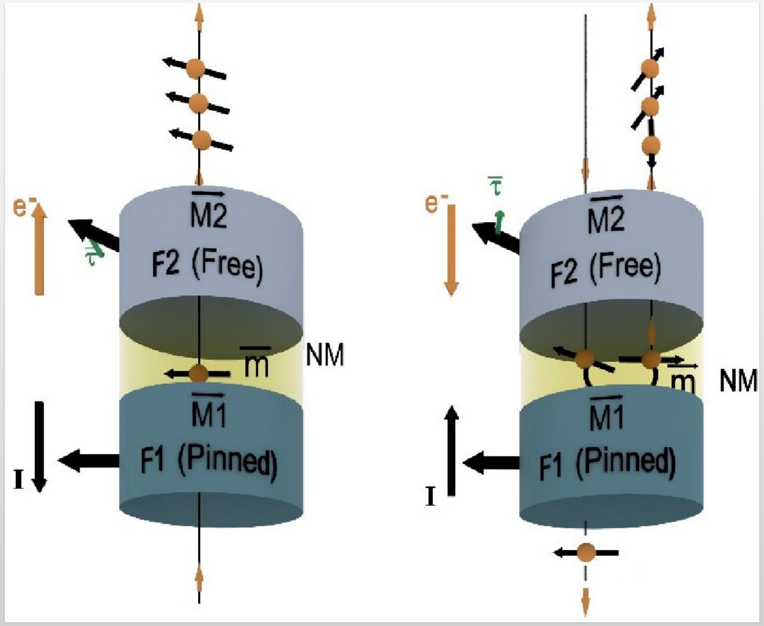
(a) CIM-Spin - ISSCC'20, Kim et al. [47]



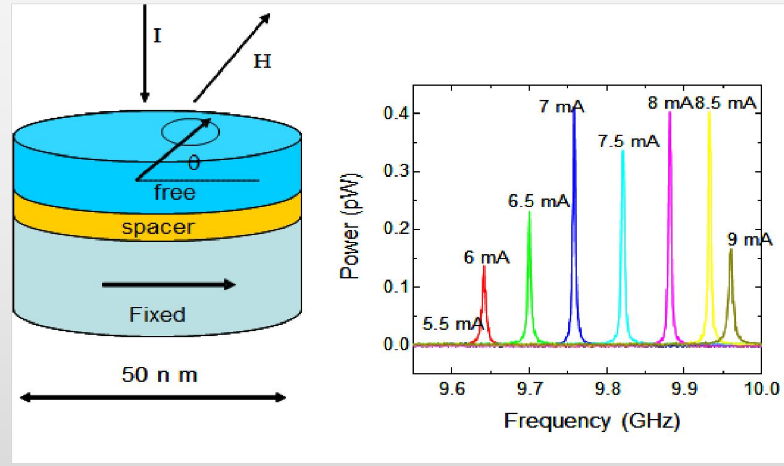




Magnetic Memory, and Oscillators

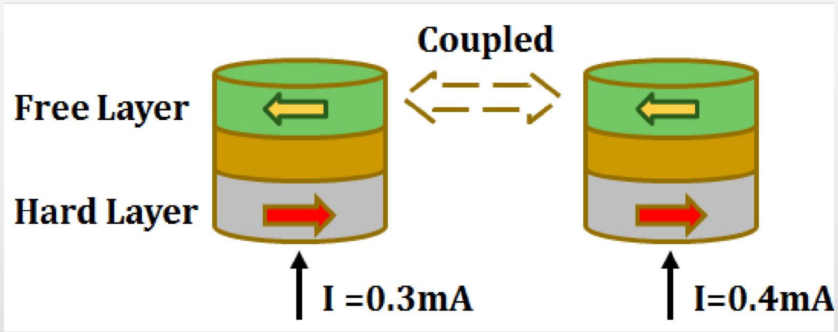


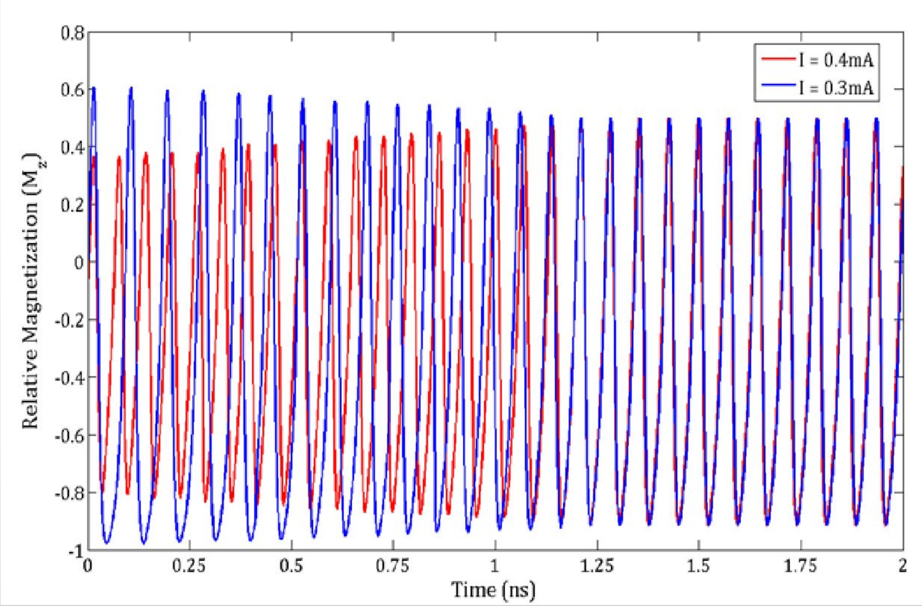
The "free" magnet aligns itself towards a preferred direction determined by the injected current



The "free" magnet oscillates about an effective field (analogous to a top precessing in a gravitational field)

Coupled Nano Oscillators





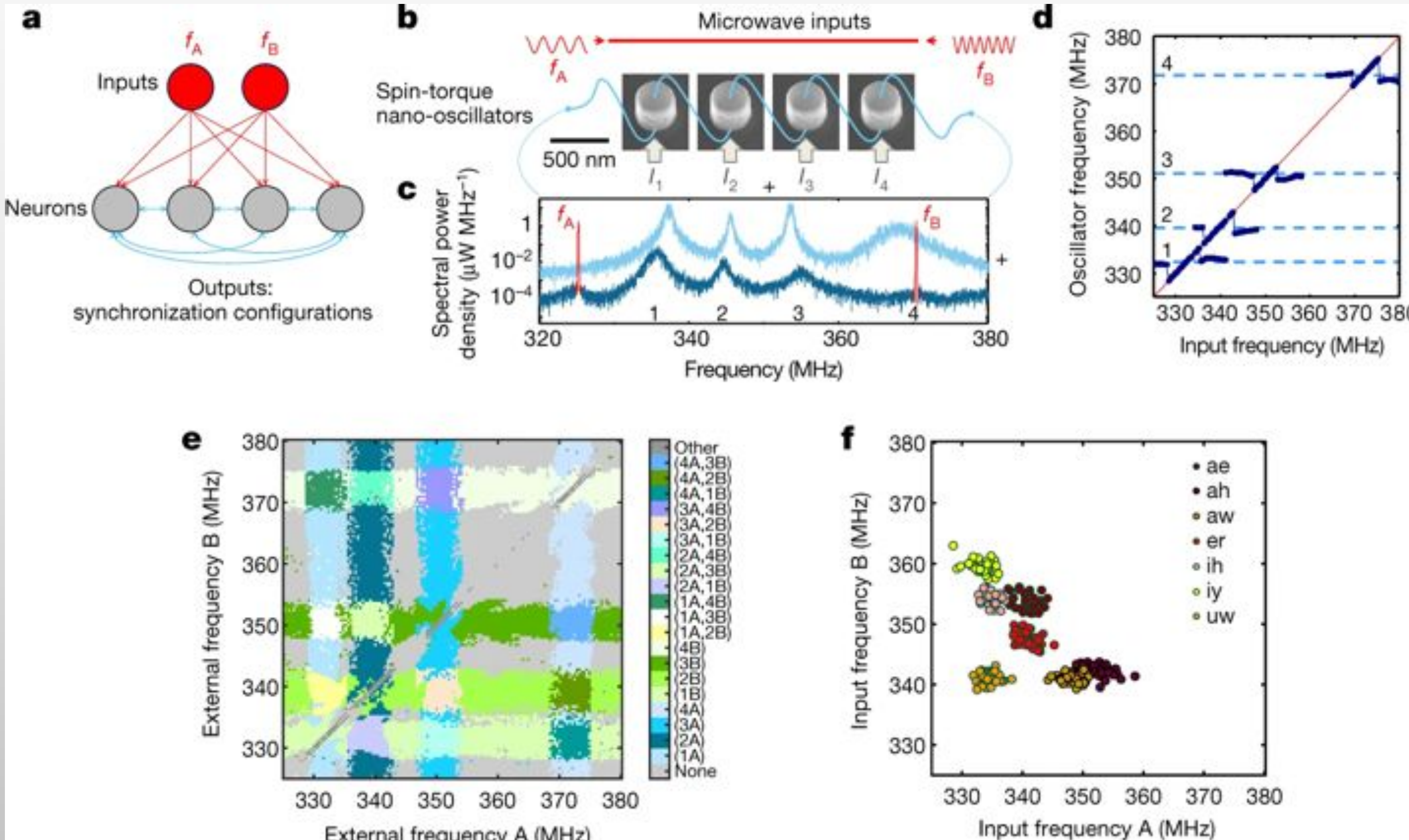
- Arrays of oscillators can lock together
- How do we change the coupling?

Nano-Patterned Coupled Spin Torque Nano Oscillator (STNO) Arrays – a Potentially Disruptive Multipurpose Nanotechnology

Mircea R. Stan, Mehdi Kabir
ECE Dept., University of Virginia
Charlottesville, VA, USA

Jiwei Lu, Stuart Wolf
MSE Dept., University of Virginia
Charlottesville, VA, USA

State of the Art



- Can do weighted oscillator networks – vowel recognition
- Romera et al, Nature 563, 2018

Acknowledgements

- Gautham, Parth, Vikram and Vignesh,
- Dr. Nitin Chandrachoodan, IITM

- Dr. Aaron Danner, Prof. Hyunsoo Yang, NUS

- Prof. Sridhar Tayur, CMU

- Centre for Quantum Information, Communication and Computing (quantum.iitm.ac.in)